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1979 J. Phys. A: Math. Gen. 12 375

(http://iopscience.iop.org/0305-4470/12/3/013)

J. Phys. A: Math. Gen., Vol. 12, No. 3, 1979. Printed in Great Britain

General relativity and conformal invariance: II Non-existence of black holes

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Received 31 January 1978, in final form 1 September 1978

Abstract. In general relativity there are certain singularities, like the initial singularity of the Friedmann cosmologies, which are due to the presence in the metric of a conformal factor which becomes zero. Using the version of the general relativity field equations produced in paper I, it is shown that there is such a singularity outside the horizon of a Schwarzchild black hole. This means that the horizon, and its interior, are mathematical constructs which have no physical meaning.

1. Conformal singularities

It was argued in a previous paper (Suggett 1979, to be referred to as I) that invariance under conformal transformations has to be regarded as an integral part of general relativity in just the same way as invariance under coordinate transformations.

When we actually attempt to solve any particular problem we have to choose a coordinate frame in which to work. In just the same way we also have to choose a conformal frame.

Singularities may be introduced into the solution by a bad choice of coordinate system. In the same way a bad choice of conformal frame may also introduce singularities.

Consider, for instance, the following situation. Suppose we choose a particular conformal frame and obtain a solution which, by suitable coordinate transformations, can be put in the form:

$$\mathrm{d}s^2 = S^2(\xi)\,\mathrm{d}\hat{s}^2 \qquad 0<\xi<\infty \tag{1}$$

where the coordinate ξ is such that $S(\xi)\to 0$ as $\xi\to 0$ and the metric $\mathrm{d}\hat{s}^2$ is defined for a larger range of ξ, i.e. $-K<\xi<\infty$ where K is some positive constant or ∞.

This situation could be looked at in either of two ways:

(i) We could say that it is the metric $\mathrm{d}s$ which has physical significance together with its singularity at $\xi = 0$, and that the metric $\mathrm{d}\hat{s}$ is derived from it by a singular conformal transformation, which cancels out the singularity in the metric and thus allows it to be extended. In this point of view the extension has *no physical significance.*

(ii) On the other hand, we could argue that the metric $\mathrm{d}\hat{s}$ is the one that has physical significance and that $\mathrm{d}s$ is obtained from it by a singular transformation which creates an unphysical singularity and also cuts off part of the space. A classic example of this

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0305-4470/79/030375+07$01.00

situation is in cosmology, where the Robertson–Walker metrics are solutions of the general relativity field equations (in the conformal frame where particle masses are constant—which we called in I frame A) and can be written in the form

$$ds^2 = S^2(T)\, ds^2_{\text{static}} \tag{2}$$

where $S(T) = 0$ for $T = 0$. In this case the conventional point of view would be (i) above in which the singularity is regarded as real, and only the region $T > 0$ of ds^2_{static} has physical significance. Hoyle and Narlikar have however argued for the other point of view—that ds^2_{static} in its full extension should be taken as physical and the singularity as unphysical. (Hoyle and Narlikar 1972a, b, c, 1974, Hoyle 1973, 1975, Narlikar 1977, Narlikar and Kembhavi 1977).

We shall show in this paper that a similar sort of behaviour occurs near a black hole. We shall demonstrate that in Islam's frame B, described in I, the exterior metric for a spherically symmetric body takes the form

$$\left.\begin{aligned} &ds^2 = S^2(r)\, ds^2_{(\text{Schwarzchild})} \\ \text{where}\quad &S(r) = 0 \qquad \text{at } r = r_0 \end{aligned}\right\} \tag{3}$$

and the radius $r = r_0$ is *outside the horizon*.

In this case the conventional point of view would be given by (ii) above—that frame B is unphysical, that the singularity it predicts is also unphysical, and that it is legitimate to extend physical space to the region $r < r_0$ to obtain the horizon and the central singularity of the usual Schwarzchild solution.

The point of view taken in this paper is that (i) is correct, not only for cosmology, but in the black hole situation as well. The places where $S \to 0$ are real singularities, and the metric cannot be extended beyond them (as mentioned above this is the conventional point of view in cosmology, but Hoyle and Narlikar have taken the opposite viewpoint. For a critical account of their ideas see Suggett (1976)). If one accepts that the metric should not be extended beyond the $S = 0$ surface then the conventional idea of a black hole is non-physical, since the horizon is in the region beyond the edge of physical space–time.

We shall define a singularity, like those discussed here, which can be removed by a suitable (if unallowable) choice of conformal frame, to be a conformal singularity. The 'big bang' is one such singularity, and as we shall show, the true end-state of gravitational collapse is another. One might possibly conjecture that all space–time singularities have this form (cf Narlikar and Kembhavi 1977).

2. Allowable frames

How, in a situation like this, is one to decide whether to accept point of view (i) or (ii)? The answer is that one must decide which of the conformal frames in question is an *allowable* frame.

An allowable frame was defined in I as a conformal frame associated with a choice of length unit which never becomes zero or infinite. This is a reasonable requirement for a length unit, since nothing can be measured against a standard which is itself vanishingly small or infinitely large, and it thus seems reasonable to regard conformal frames associated with length units that break this condition as being non-allowable.

General relativity is usually written in conformal frame A, which is defined by the requirement that particle masses are always constant. However, it was shown in I that for localised distributions of matter there is some doubt about the allowability of frame A, and that in order to discuss such problems it is better to use Islam's frame B, defined by the requirement that when the mass of a local particle is split into parts which can be regarded as being generated locally, and as being due to non-local interactions respectively it is the non-local part that is constant. This means that in this frame a local problem can be solved in purely local terms without any explicit interaction from non-local sources.

Frame B is allowable, and we shall show below that the metric round a spherical object in frame B is

$$\left.\begin{aligned} ds^2 &= S^2(r)\, ds^2_{(\text{Schwarzchild})} \\ \text{where} \qquad S(r) &= 1 - \frac{K}{3}\log\left(\frac{1+m/2r}{1-m/2r}\right) \end{aligned}\right\} \tag{4}$$

r being the isotropic radial coordinate of the Schwarzchild metric. $S(r)$ has a zero at a radius $r = r_0$ which we shall show is *outside* the horizon of the Schwarzchild metric. Thus in accordance with what we said above, frame A *is* non-allowable and the parts of the frame A manifold with $r < r_0$ are non-physical. This includes the horizon.

3. Proof that the Schwarzchild black hole is unphysical

We are looking for the metric in frame B which is conformal to the spherically symmetric Schwarzchild solution in frame A. Let us write the metric in frame B in *isotropic* coordinates.

$$ds^2 = e^{\nu}\, dt^2 - e^{\lambda}(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2)$$

(These coordinates are particularly useful in this context because, under a conformal transformation, they retain the property of being isotropic—whereas the radial 'area' coordinate, defined such that $4\pi\rho^2$ is the surface area of a sphere of coordinate radius ρ in which the Schwarzchild metric is usually written, is different in different conformal frames.)

The conformal transformation which takes us from frame B to frame A is given by

$$\left.\begin{aligned} ds^2_{\text{A}} &= \Lambda^2\, ds^2_{\text{B}} \\ \text{where} \qquad \Lambda &= 1 + \sigma/\sigma_0 = 1 + X \end{aligned}\right\} \tag{5}$$

where we denote σ/σ_0 by X. So, in frame A,

$$ds^2 = e^{\nu}(1+X)^2\, dt^2 - e^{\lambda}(1+X)^2(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2) \tag{6}$$

which must therefore be the Schwarzchild metric in isotropic co-ordinates:

$$ds^2 = \left(\frac{1-m/2r}{1+m/2r}\right)^2 dt^2 - \left(1 + \frac{m}{2r}\right)^4 (dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2). \tag{7}$$

So we must make the following identifications for the metric coefficients:

$$e^{\nu}=\left(\frac{1-m/2r}{1+m/2r}\right)^2\frac{1}{(1+X)^2}\qquad e^{\lambda}=\frac{(1+m/2r)^4}{(1+X)^2}. \tag{8}$$

We thus see that the metric in frame B is given by

$$\left.\begin{aligned}&ds^2=S^2(r)\,ds^2_{(\text{Schwarzchild})}\\ \text{where}\qquad &S^2(r)=1/(1+X)^2.\end{aligned}\right\} \tag{9}$$

So to determine S we need to obtain X, which as derived in I, satisfies in frame B in vacuo

$$\Box^2 X=0. \tag{10}$$

After a first integration we obtain in the isotropic co-ordinates

$$r^2\exp[\tfrac{1}{2}(\nu+\lambda)]\,dX/dr=\text{constant}. \tag{11}$$

We shall show in the next section that the constant has to be negative, and for reasons also explained below we shall therefore write it as

$$\text{constant}=-Km/3\qquad K>0 \tag{12}$$

so that

$$r^2\exp[\tfrac{1}{2}(\nu+\lambda)]\,dX/dr=-Km/3 \tag{13}$$

where ν and λ are given by equation (8), i.e.

$$\frac{r^2(1-m^2/4r^2)}{(1+X)^2}\frac{dX}{dr}=-\frac{Km}{3}. \tag{14}$$

Equation (14) can easily be integrated and, using the boundary condition $X\to 0$ at $r\to\infty$, leads to

$$1-\frac{1}{1+X}\frac{K}{3}\log\left|\frac{1+m/2r}{1-m/2r}\right| \tag{15}$$

i.e.

$$S(r)=\frac{1}{1+X}=1-\frac{K}{3}\log\left|\frac{1+m/2r}{1-m/2r}\right| \tag{16}$$

so we have, in frame B,

$$\begin{aligned}&ds^2=S^2(r)\,ds^2_{(\text{Schwarzchild})}\\ &1+\sigma/\sigma_0=S^{-1}(r)\end{aligned} \tag{17}$$

with S given by equation (16). This metric has a conformal singularity at the radius $r=r_0$ where $S(r_0)=0$, i.e.

$$1-(K/3)\log|(1+m/2r_0)/(1-m/2r_0)|=0$$

i.e.

$$r_0=(m/2)\frac{\exp(3/K)+1}{\exp(3/K)-1}. \tag{18}$$

In terms of the Schwarzchild area coordinate $\hat{\rho}$ defined by

$$\hat{\rho} = r(1 + m/2r)^2 \tag{19}$$

we have a conformal singularity at

$$\hat{\rho} = \hat{\rho}_0 = 2m \frac{\exp(6/K)}{\exp(6/K) - 1} > 2m. \tag{20}$$

In other words the conformal singularity is outside the horizon, which thus becomes unphysical.

4. The value of *K*

The arbitrary constant in equation (11) which we have denoted by $-Km/3$ arises because we are solving the vacuum equation (10). K can be evaluated if we introduce an explicit source into the problem. For instance if we take the source to be a 'point particle' to which we assume the Newtonian approximation can be applied, we obtain, by comparing the asymptotic behaviour of equation (11) with the Newtonian approximation obtained in I, that the constant has the value $-m/3$, thus making $K = 1$ (which was why we chose the constant in this form). In a previous discussion of the black hole problem in frame B (Suggett 1975, reprinted with minor changes as Appendix I of Suggett 1976) it was assumed that the result of the collapse of a large body was such a Newtonian object, so K was taken equal to 1 and this was called the *Newtonian boundary condition.*

However, it was pointed out by Dr B F Schutz (private communication) that the result of gravitational collapse need not satisfy the Newtonian boundary condition and that it should be possible to obtain a value for K by following the dynamics of the actual collapse. Preliminary investigations along these lines have, however, not been very helpful, since the collapsing body runs into a conformal singularity whilst still in a dynamic state, and after the singularity forms the space–time loses its predictability.

Fortunately, to be able to discuss the possibilities for the static end states we do not need an explicit value for K. All we need to obtain the conformal singularity deduced above is the result that $K > 0$.

The case $K < 0$ can be excluded immediately, since by equation (13) this would imply that X, and hence σ, is increasing for large radii. However, this is incompatible with our assumptions:

(i) that mass is positive, (21*a*)
(ii) that the local matter is dynamically *insular*, $\sigma \to 0$ at ∞. (21*b*)

The possibility $K = 0$ is more interesting, since by equation (13) we have $X = \text{constant}$, and hence by assumption (21*b*) $X = 0$. This means that the object generates *no* mass field itself, in which case the total mass field is non-locally generated and hence frames A and B become identical, so that the solution in frame B is also the Schwarzchild black hole. Is it possible that an object resulting from gravitational collapse should generate no mass field (i.e. have no inertial charge, and hence no inertial mass)? The answer yes would perhaps be expected on the basis of Price's theorem ('a black hole has no hair') which shows how scalar fields are radiated away during collapse. However, as mentioned above, the actual collapse in frame B appears to run into an unpredictable situation, and hence Price's theorem is inoperative. The answer is in fact no, as can be seen very simply by the following argument. If it were true that the object generated no mass field then frame A and frame B would be identical and the Schwarzchild solution would be a valid solution in frame B. However, as was pointed out to me by J V Narlikar (private communication; the point is also discussed briefly in a footnote in Hoyle and Narlikar 1966) the Schwarzchild solution can formally be regarded as a solution of Einstein's equations with matter density

$$\rho = M\delta(\boldsymbol{r}) \qquad GM = m$$

and hence scalar curvature

$$R = 8\pi GM\delta(\boldsymbol{r}) \tag{22}$$

but in frame B, equation (14) of I shows that R is *identically* zero. Hence the Schwarzchild solution cannot be a frame B solution and hence $K \neq 0$. We must therefore have $K > 0$.

5. The final outcome of collapse

Thus we see that the only possible solution in frame B for the exterior field of a compact spherical distribution of matter is of the form given by equations (16) and (17) with $K > 0$.

This solution, representing a conformal singularity, must therefore represent the final outcome of gravitational collapse. The singularity is not hidden by a horizon, since the predicted radius of any possible horizon is in the region of space which is made 'non-physical' by the existence of the conformal singularity.

6. Conclusion

The Schwarzchild black hole has been shown to be merely a mathematical construct. The edge of the space–time to which our present theories are applicable falls outside the radius at which the horizon would be.

Similar proofs can be given for the Kerr and Kerr–Newman black holes, although the extra variable makes the calculations more complicated (details are given, but with the assumption of the Newtonian boundary condition, in Suggett 1975, 1976).

Acknowledgment

The work described in this paper was carried out while the author was Tutorial Fellow in the Department of Applied Mathematics and Astronomy at University College, Cardiff. The present form of the paper owes much to conversations held at that time with Dr B F Schutz.

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